

The Cost of Concreteness in Transfer of Mathematical Knowledge

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Kaminski

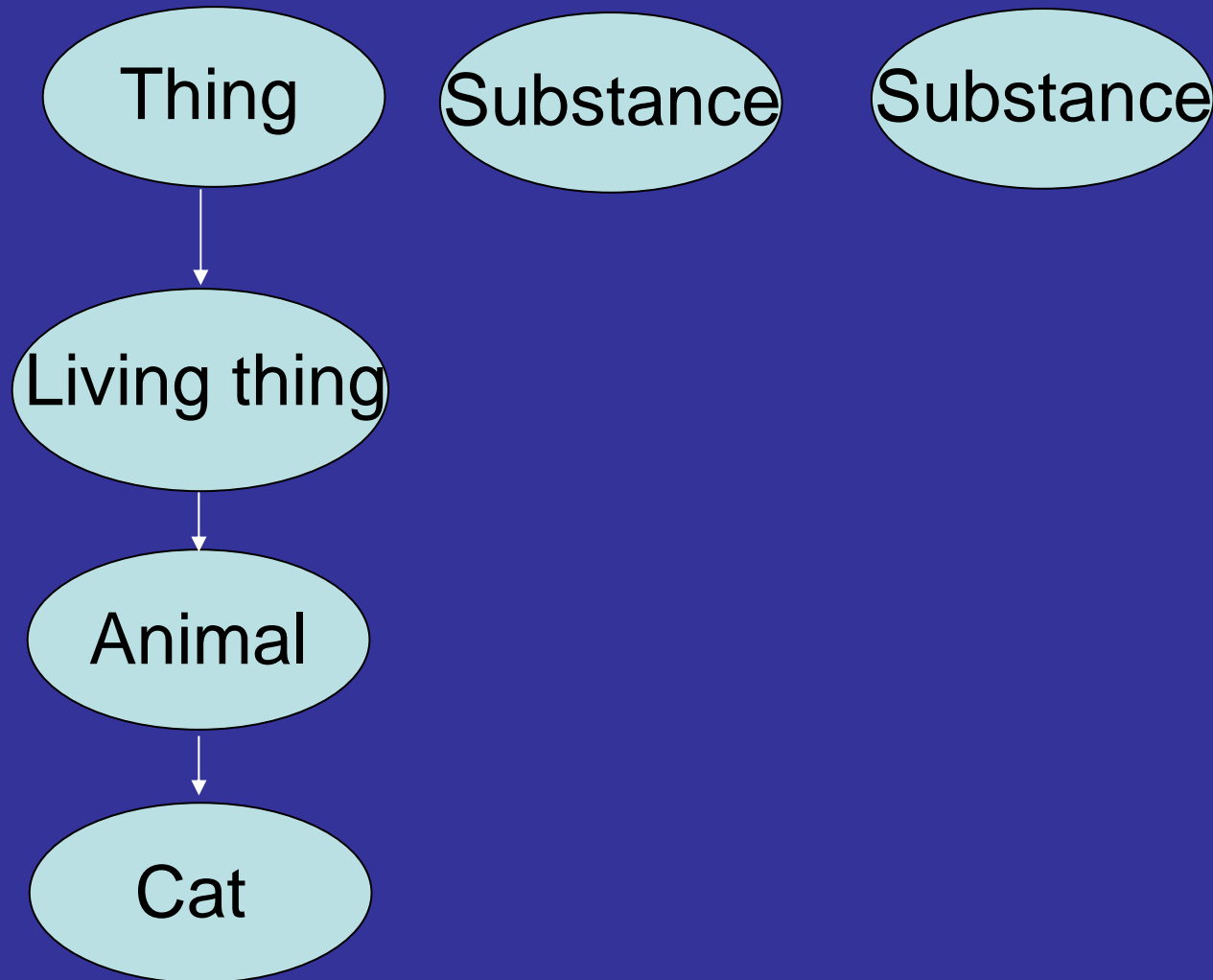
Center for Cognitive Science
Ohio State University

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Overview of Talk

1. Concrete vs. Abstract Concepts
2. Concrete vs. Abstract Instantiations
3. Effects of concrete instantiations on learning and transfer
4. Potential mechanisms underlying effect of concreteness
5. Benefits and costs of abstract instantiations

Concrete vs. Abstract Concepts I: Place in Ontology



Concrete vs. Abstract II: Physicality

- “Cat” vs. “Happiness”

Concrete vs. Abstract III: Object vs. Relational Concepts

Object Concepts

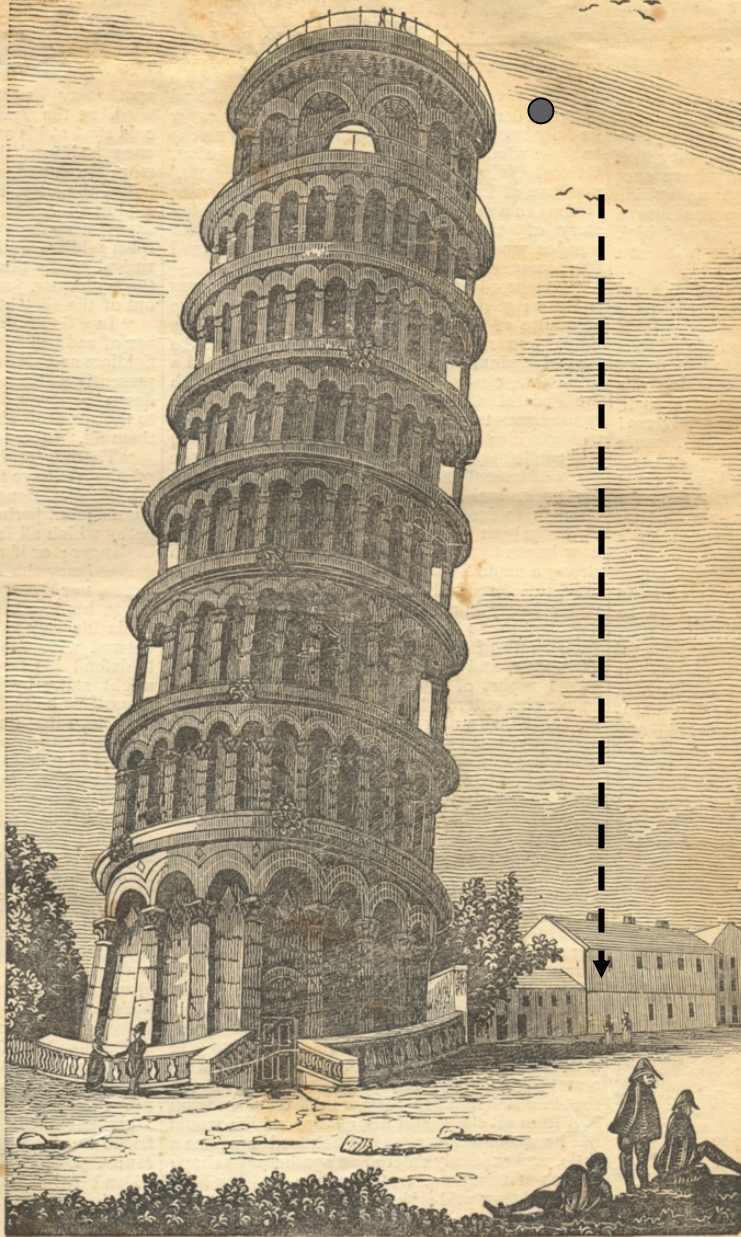
Cats



Bears



Relational Concepts



"LEANING TOWER OF PISA,"

Concrete Concepts

- Concrete concepts are grounded in similarity
- They can be acquired spontaneously

Abstract Concepts

- Abstract concepts are independent of similarity.
- And they are unlikely to be acquired spontaneously.
- Most of mathematical and scientific concepts are abstract.

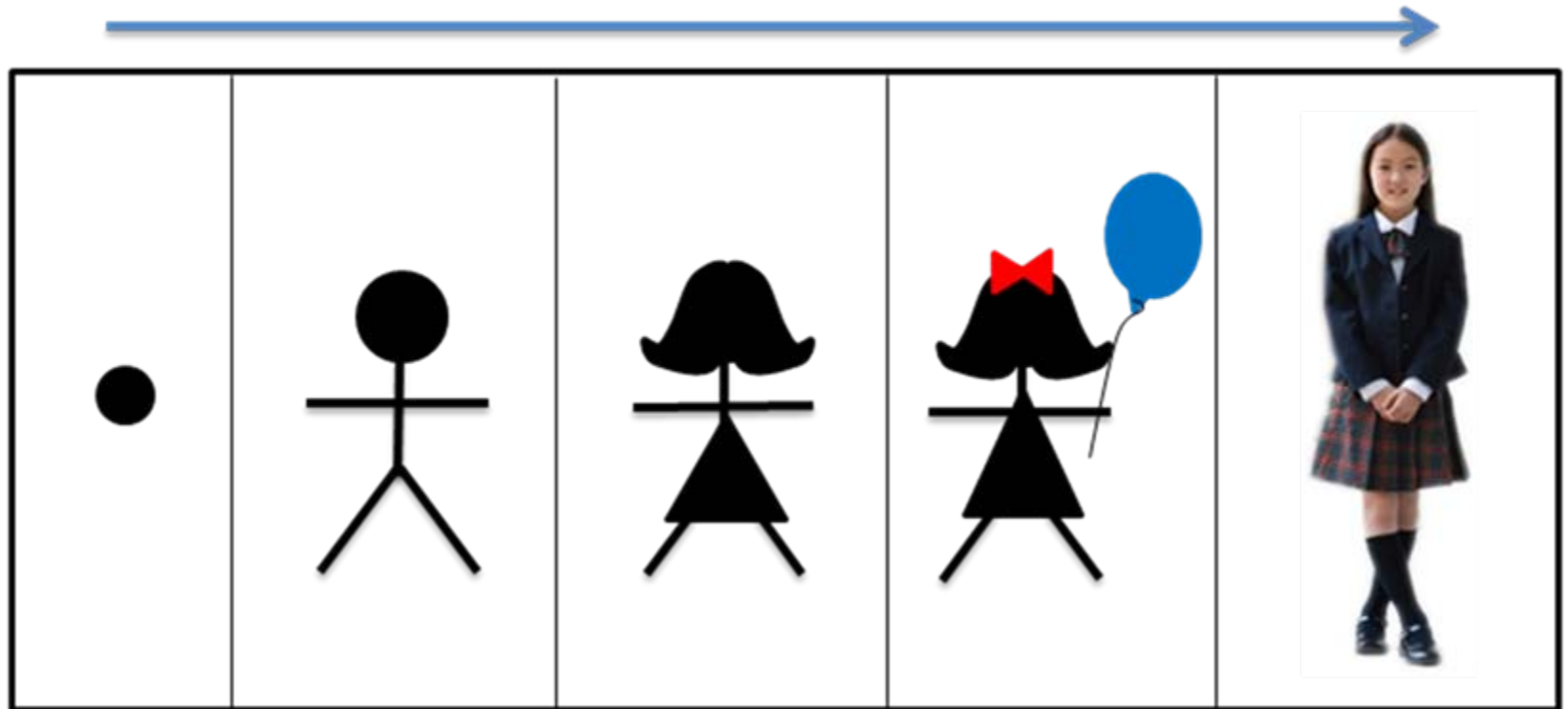
In this research we focus on
abstract mathematical concepts

Concrete vs. Abstract Instantiations

Concrete instantiations communicate more
information than abstract ones

Possible symbols for the concept “*Individual*”

Increasing information/ Increasing concreteness



Increase in concreteness decreases the “referential breadth”

Concrete Instantiations

Physical manipulatives

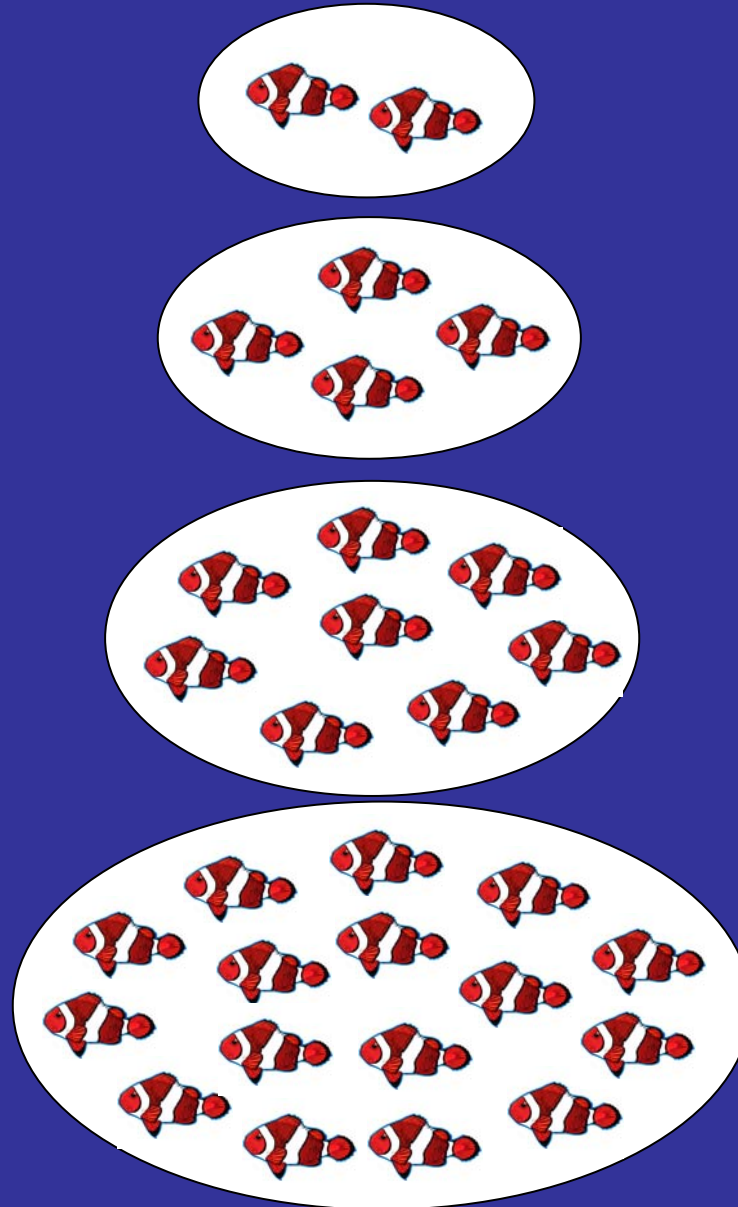
Detailed Pictures

Contextualized story problems

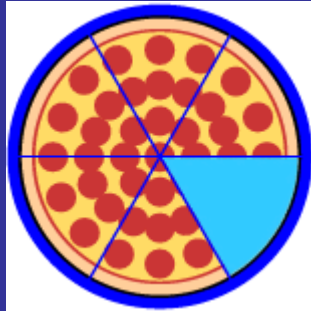
Promising:

- Engaging for students

Exponential Growth:



Fractions



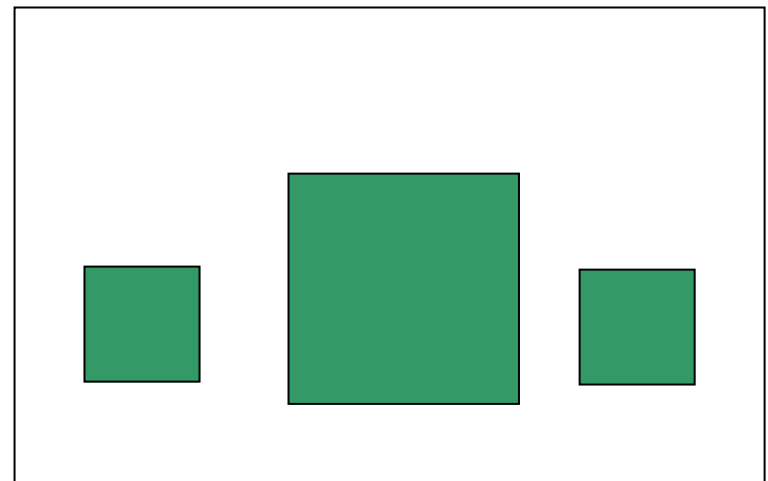
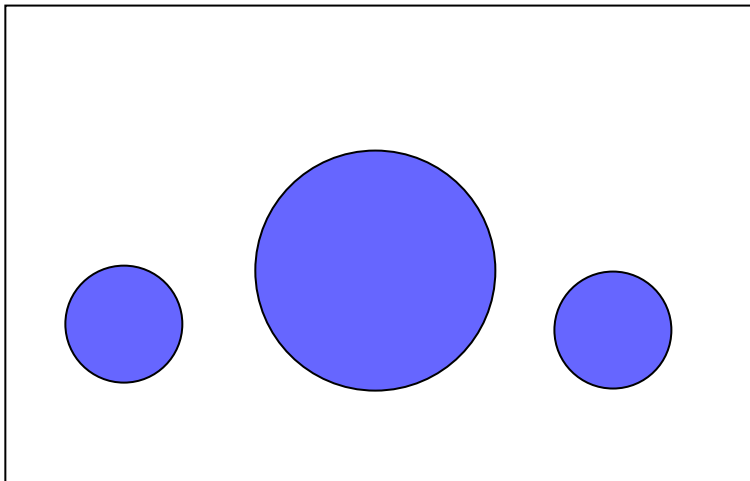
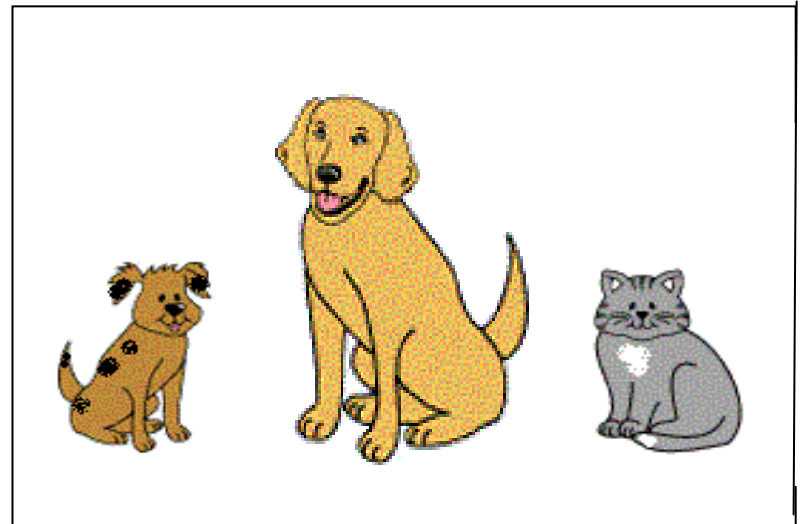
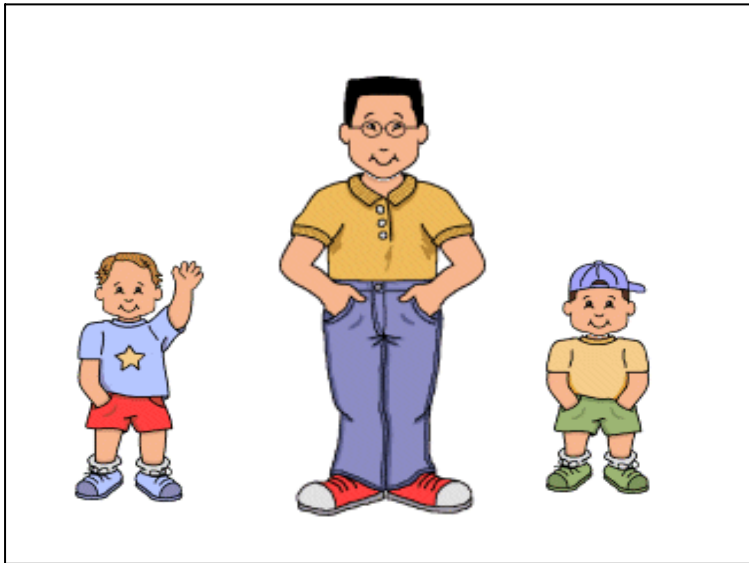
Reasons for skepticism: Concreteness may hinder transfer

- More transfer from algebra to physics than the reverse (Bassok & Holyoak, 1989).
- Transfer to a novel instantiation is often worse when participants learn one or more concrete instantiation
 - Goldstone & Sakamoto (2003)
 - Son, Goldstone, & Smith (in press)
 - Sloutsky, Kaminski, & Heckler (2005)
 - Kaminski, Sloutsky, & Heckler (2008)

Why?

1. Superficial features can compete for attention with the relational structure. (Goldstone & Sakamoto, 2003)
2. Relational structure is less likely to be noticed between perceptually rich domains than perceptually sparse domains. (Gentner & Medina, 1998; Markman & Gentner, 1993) ^{*}
—

What is the common relation?



Why?

1. Superficial features can compete with relational structure for attention. (Goldstone & Sakamoto, 2003)
2. Relational structure is less likely to be noticed between perceptually rich domains than perceptually sparse domains. (Gentner & Medina, 1998; Markman & Gentner, 1993)
3. Irrelevant information can be misinterpreted as part of the relational structure. (Bassok, Wu & Olseth, 1995)
4. Concrete objects make poor symbols. (DeLoache, 2000; Schwartz, 1995).
5. All these ideas suggest that concreteness may hinder mapping of a learning domain onto a transfer domain.

This idea has been tested in a
number of experiments

Commutative Group of Order 3

- Set with 3 Elements
- Associated Operation
- Properties
 - Identity Element
 - Associativity
 - Commutativity
 - Inverses
- Addition Modulo 3

Idea of Addition Mod 3

- 0 1 2
- $0 + 1 = 1$
- $0 + 2 = 2$
- $1 + 1 = 2$
- $1 + 2 = 0$

Instantiations

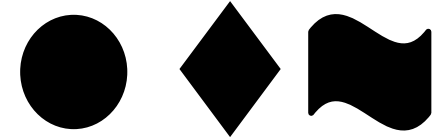
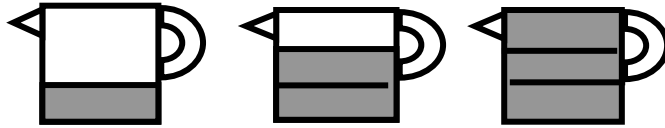
Abstract / Generic – like mathematics:
symbolic language

Concrete

Concrete

Generic

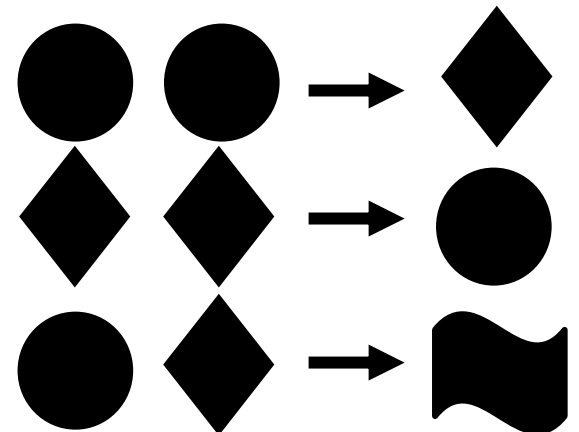
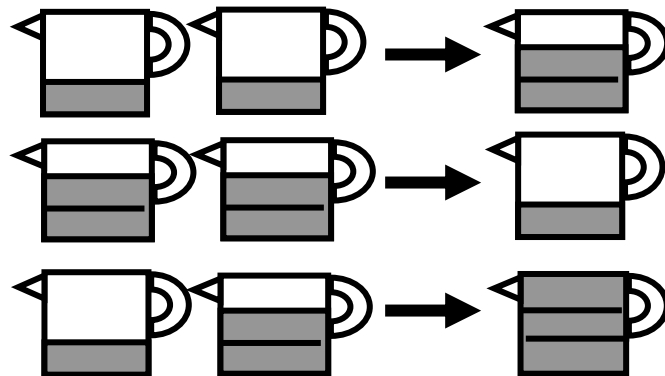
Elements:



Identity:



Specific Rules:



Transfer Domain

Children's game

Elements:



Identity:



Specific Rules:

If kids point to:



Winner points to:



Experiment 1

Conditions: Concrete and Abstract

Subjects: Undergraduate students

Study Phase (presentation of rules, several examples, questions with feedback)


Test Phase

Examples of Learning Questions

1. Find the resulting symbol:  → _____









Choose: 1.)  2.)  3.) 

2. Find the resulting symbol:  → _____

Choose: 1.)  2.)  3.) 

3. What symbols go in the blanks to make a correct statement?

____ , ____ ,  → 

Choose: 1.)  and  2.)  and 
 3.)  and  4.)  and 

Examples of Transfer Questions

1. What object do you think the winner will point to when the other kids point

to  then  ?

Choose:

1.)




2.)



3.)



2. What object does the winner point to when the other kids point to  then

 then  ?

Choose:

1.)



2.)




3.)



3. If a group of kids wants the winner to point to what other objects do they need to point to?



, and they first point to  ,

Choose:

1.)



and



2.)



and



3.)



and



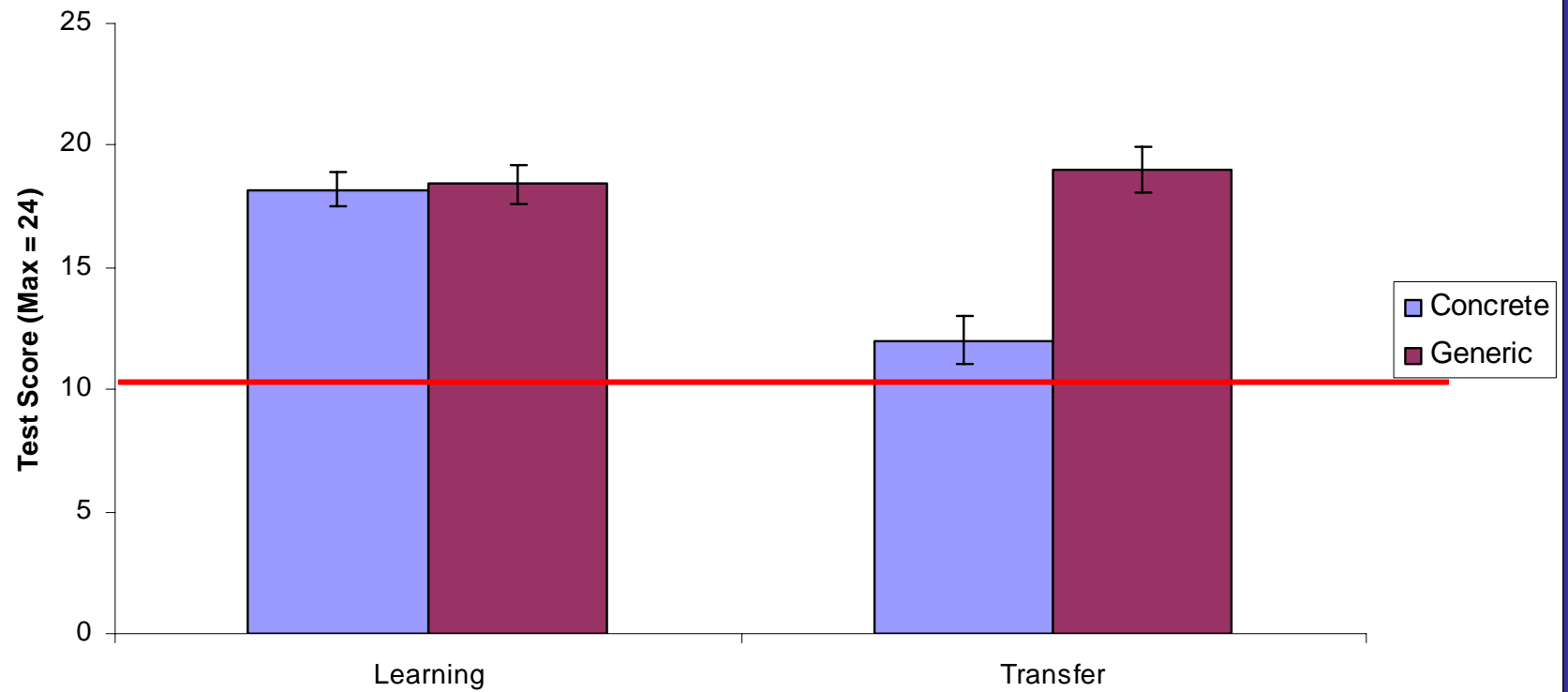
4.)



and



Results of Experiment 1



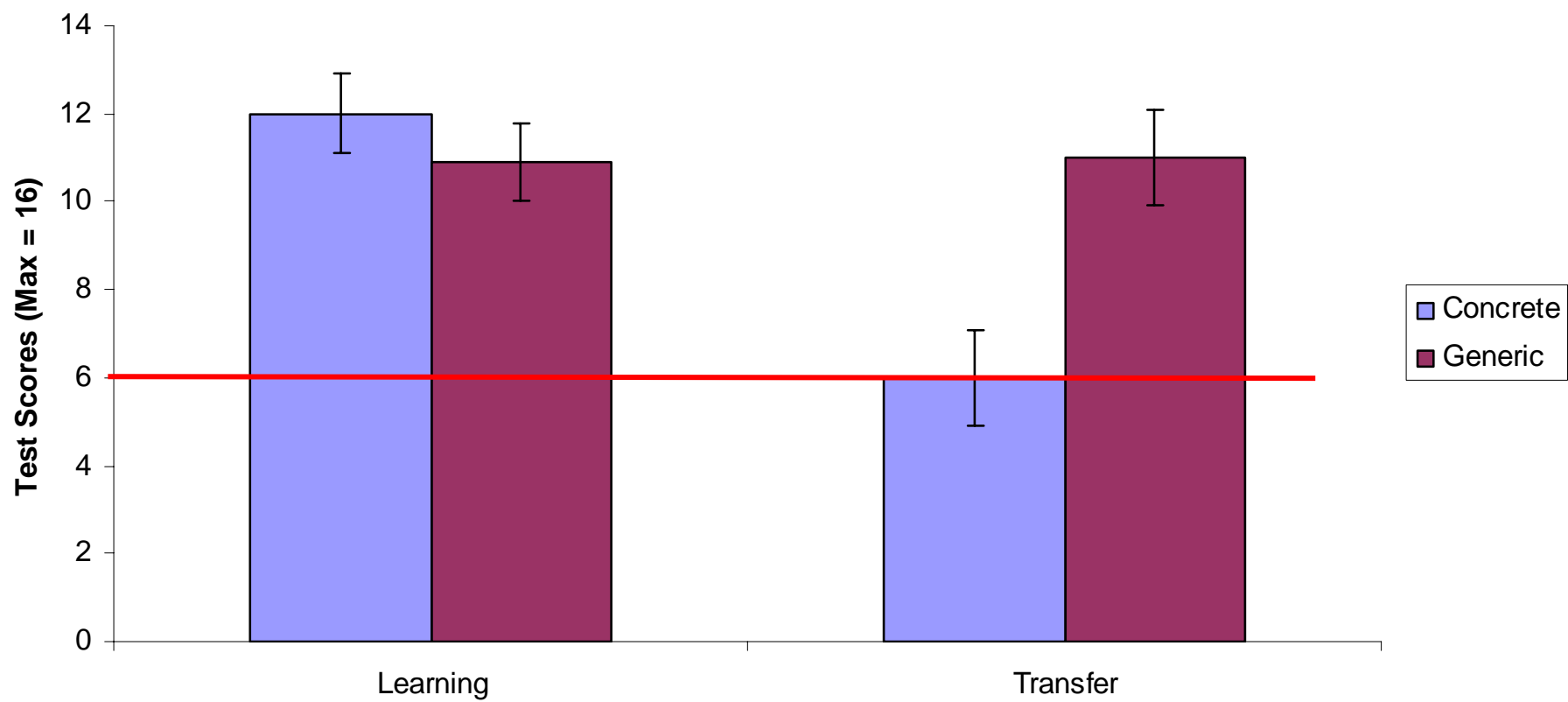
Mapping

- 100% of the participants in the Abstract condition correctly matched elements from the learning instantiation to the transfer instantiation
- Only 25% of the Concrete condition did so.

Experiment 2: Younger Participants

6th-7th graders

Results of Experiment 2



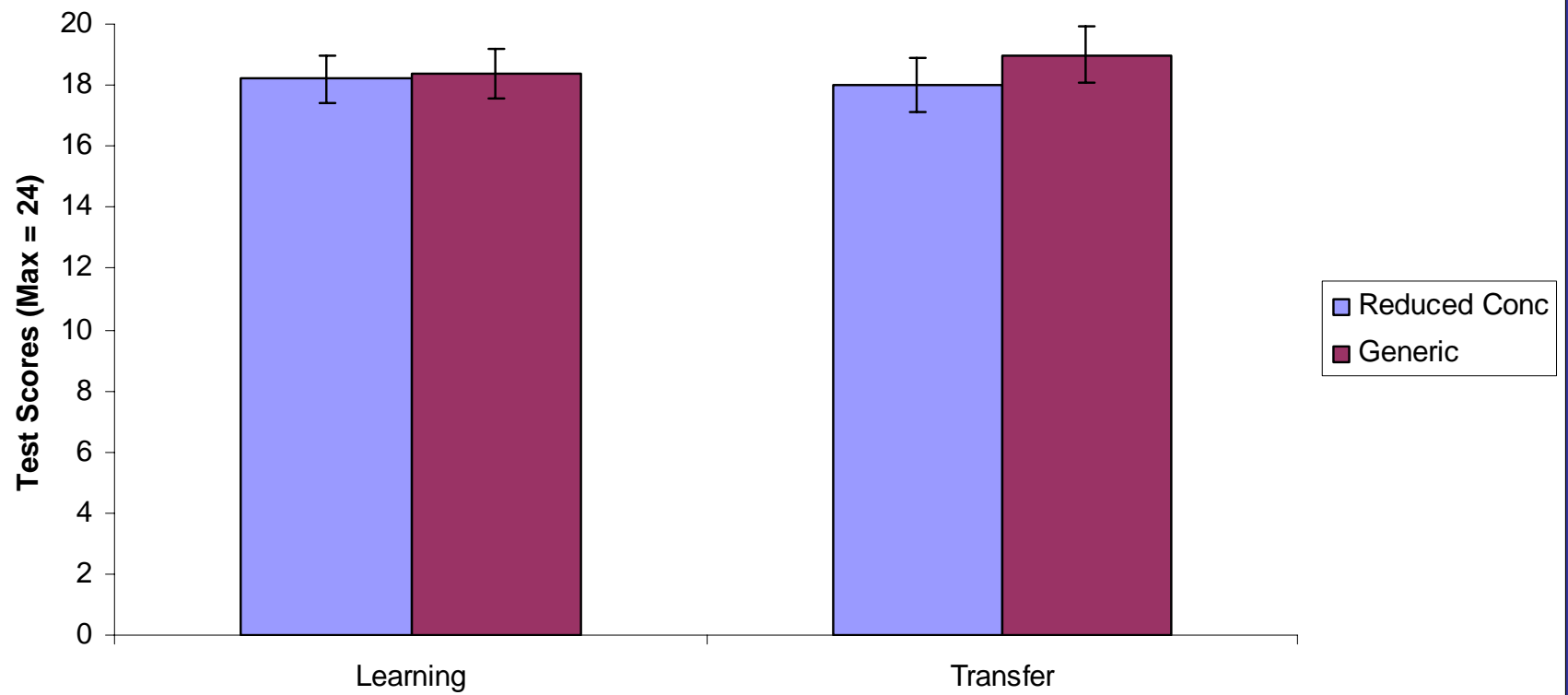
Mapping

- 75% of the participants in the Abstract condition correctly matched elements from the learning instantiation to the transfer instantiation
- Only 20% of the Concrete condition did so.

Experiment 3: Eliminating Concreteness

In the Concrete condition, the
storyline was removed

Results of Experiment 3



Mapping

- 88% of the participants in the Abstract condition correctly matched elements from the learning instantiation to the transfer instantiation
- 85% of the Reduced Concrete condition did so.

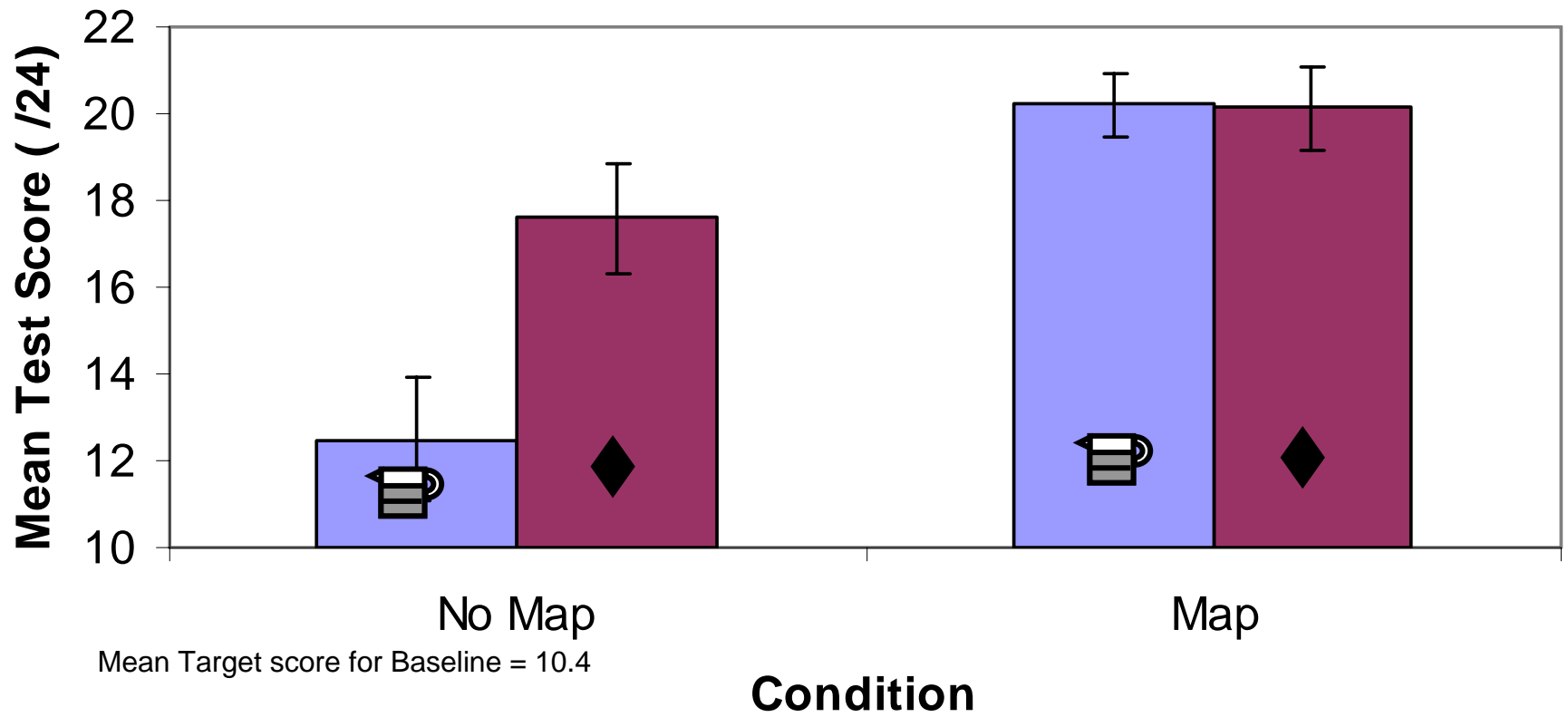
Experiment 4: Direct Test of the mapping hypothesis

Undergraduate students

Phase 1: Training and Testing in the Learning Domain

Phase 2: Present Transfer Domain
+ give participants mapping of elements of the Learning Domain onto the Transfer Domain

Mean Test Score in Transfer Domain



Benefits and Costs of Abstract and Concrete Instantiations

- Concrete instantiations may hinder transfer
- Assisting in mapping cannot resolve this problem because all potential transfer domains are not known a priori
- Abstract instantiations result in better transfer, however they may potentially affect learning (that's why concrete instantiations became popular in the first place)

Conclusions

- Concreteness hinders transfer, without necessarily hindering learning
- Concreteness hinders transfer by preventing mapping between the learning and transfer domains
- If concrete material is to be used – additional steps should be taken to help learners transfer knowledge

Thank you!

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