

Interactive Support for Mathematical Reasoning and Metacognitive Judgments of Learning

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Chasm between learning sciences & educational practice

Dimensions of today's challenge

- Empirical dimension
 - Lots of rigorous principle-testing lab studies
 - Lots of realistic classroom design research
 - *Too few experiments combine both*
- Theoretical dimension
 - *Almost as many theories as there are results!*
- Practical dimension
 - Great ideas in field, ignored in lab
 - Strong scientific results being ignored in field

Pittsburgh Science of Learning Center (PSLC) Purpose Statement

Leverage cognitive theory and
cognitive modeling to identify the
instructional conditions that cause
robust student learning

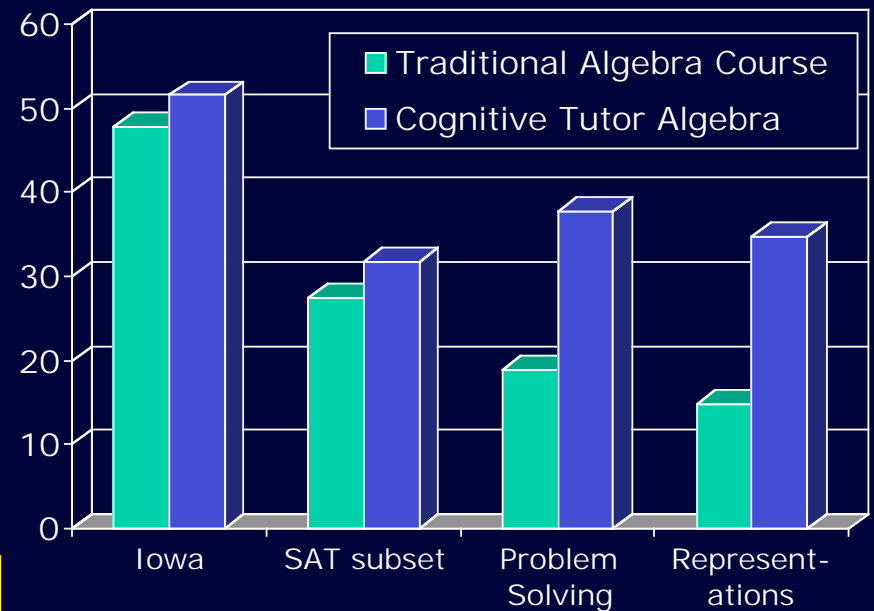
Overview

- Background: Cognitive Tutors & PSLC
- Assistance Dilemma
 - Argument for less
 - Argument for more
- Example-problem dimension --
cognitive & metacognitive issues

Real World Impact of Learning Science

Cognitive Tutor Algebra course

- Based on cognitive theory & AI models of student thinking & learning
- Most widely used & evaluated Intelligent Tutoring System
 - > 2600 schools
 - > 10 full-year field studies demonstrating better student learning



Koedinger, Anderson, Hadley, & Mark (1997).
Intelligent tutoring goes to school in the big city.

Algebra Cognitive Tutor Sample

Analyze real world problem scenarios

An experimental aircraft has sunk off the coast of South Africa at a depth of 12,790 feet. The military have located the aircraft and are in the process of raising it to the surface. It is currently 7625 feet below the surface and is being raised at the rate of 185 feet per hour. (Hint: Consider the direction above sea level to be positive)

1. How deep was the aircraft five hours ago?
2. How deep will the aircraft be five hours from now?
3. When did the military start raising the aircraft?
4. When will the aircraft reach the surface?

To write an expression, define a variable for the time from now and use this variable to write a rule for the depth of the aircraft.

Use table, spreadsheet

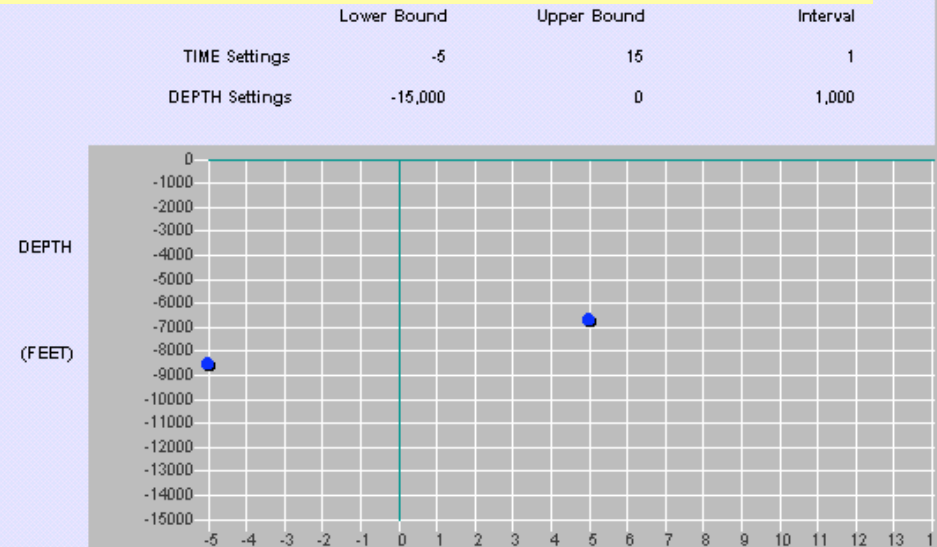
	TIME	DEPTH
Unit	HOURS	FEET
Expression	H	$-7625 + 185H$
1	-5	-8,550
2	5	-6,700
3	-27.9189...	-12,790

Model tracing to provide context-sensitive Instruction

Messages

You have entered the given 0 in the wrong column of the worksheet.

Use graphs, graphics calculator



Use equations, symbolic calculator

$$-7625 + 185H = -12790$$

Add 7625

$$185H = -5,165$$

Divide by 185

$$H = -1,033/37$$

Tracked by knowledge tracing

- ✓ Changing axis bounds
- ✓ Changing axis intervals
- Correctly placing points
- Write expression, any form
- Find Y, any form
- Find X, any form
- Identifying units
- Entering a given

Prior achievement:

Intelligent Tutoring Systems
bring learning science to schools

A key PSLC inspiration:

Educational technology as
research platform to generate
new learning science

Logic of Pittsburgh Science of Learning Center (PSLC)

- Support experimental studies that
 - Test fundamental principles, not whole courses
 - Are internally & externally valid
- Create a theory of “robust learning”
- Leverage technology & computational modeling

A Bet About the Future

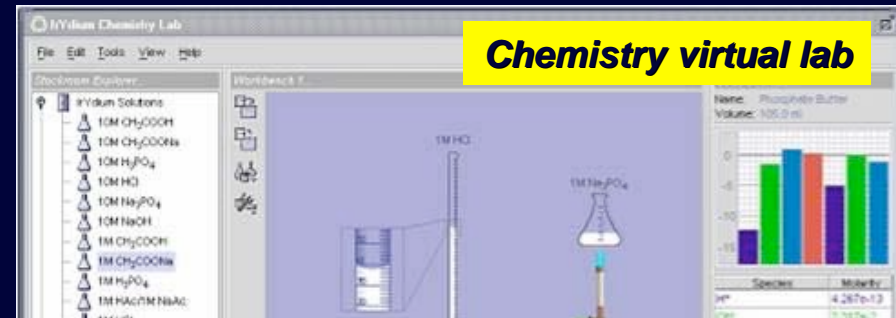
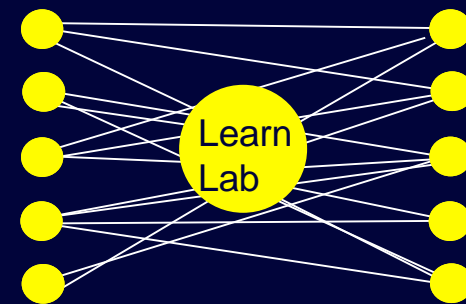
- The key to the 21st century university:
- Technology!
- And not because of the direct benefits of technology
- But because of vast data on learning & fast feedback to instructors, designers, administrators, researchers ...

LearnLab: Like a research hospital for learning

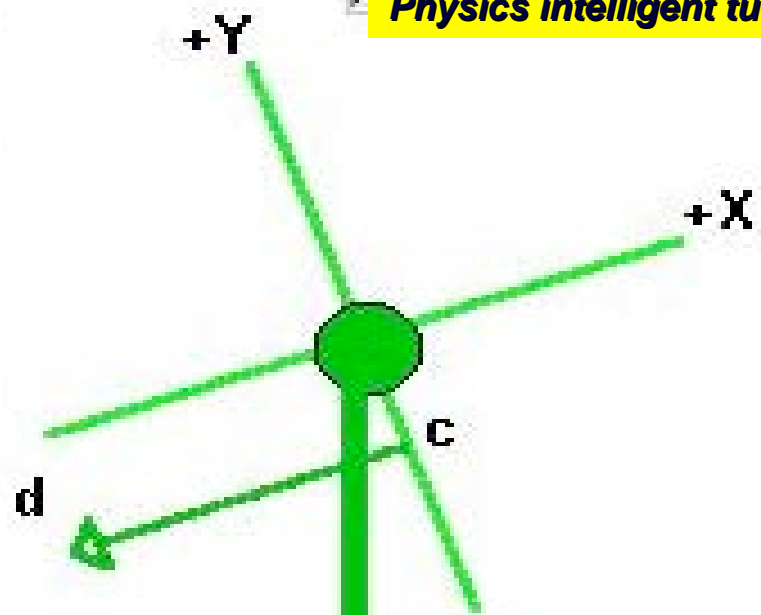
- Tech enhanced courses in Science, Math & Language
- Agreements with schools
 - *Instrument* for continuous embedded assessment
 - *In Vivo Experiments* test principles for achieving robust learning

Researchers

Schools



Physics intelligent tutor



Example *In Vivo* Experiment on "Self-Explanation"

- Self-explanation: Have students explain steps in solutions
- *In vivo* experiments: Tightly controlled principle-testing experiment embedded in a real course

Aleven, V. & Koedinger, K. R. (2002). An effective metacognitive strategy: Learning by doing and explaining with a computer-based Cognitive Tutor. *Cognitive Science*, 26(2)

Problem Solving Condition

(Ecological control: Tutor as it was)

External Angle & Parallel Lines

Given: $ON \parallel EC$. If the measure of Angle SOR is a right angle, find the measure of Angle SRN.

$m\angle SOR$
 $m\angle OSC$
 $m\angle OSR$
 $m\angle ESR$
 $m\angle SRN$

Messages

Some reasons dealing with parallel lines are highlighted in the Glossary. Which of these reasons is appropriate?

You can click on each reason in the Glossary to find out more.

Glossary

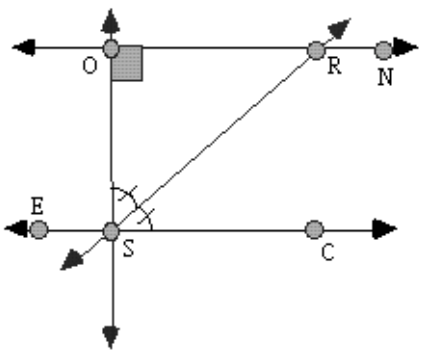
- Converse of Isosceles Triangle (Theorem)
- Isosceles Right Triangle
- Triangle Sum (Theorem)
- Linear Pair
- Linear Trio
- Parallel Lines --- Corr. Angles Are Cong.**
- Parallel Lines --- Alt. Int. Angles Are C.**
- Parallel Lines --- Alt. Ext. Angles Are C.**
- Parallel Lines --- Int. Angles on the Same Side of Transversal Are S.**

If two parallel lines are intersected by a transversal, then alternate interior angles are congruent.

Example: L_1 and L_2 are parallel lines, intersected by transversal T . $\angle 1$ and $\angle 2$ are alternate interior angles. If $m\angle 1$ is 37° , then $m\angle 2$ is also 37° .

Explanation Treatment Condition

External Angle & Parallel Lines



Given: $ON \parallel EC$. If the measure of Angle SOR is a right angle, find the measure of Angle SRN.

m<SOR	<input type="text" value="90"/>	Reason	<input type="text" value="given"/>
m<OSC	<input type="text" value="90"/>	Reason	<input type="text" value="int angles same side"/>
m<OSR	<input type="text" value="45"/>	Reason	<input type="text" value="angle bisection"/>
m<ESR	<input type="text" value="135"/>	Reason	<input type="text" value="angle addition"/>
m<SRN	<input type="text"/>	Reason	<input type="text"/>

Messages

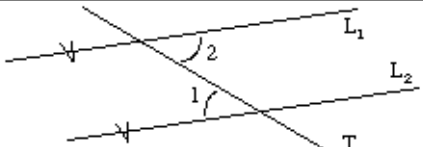
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Glossary

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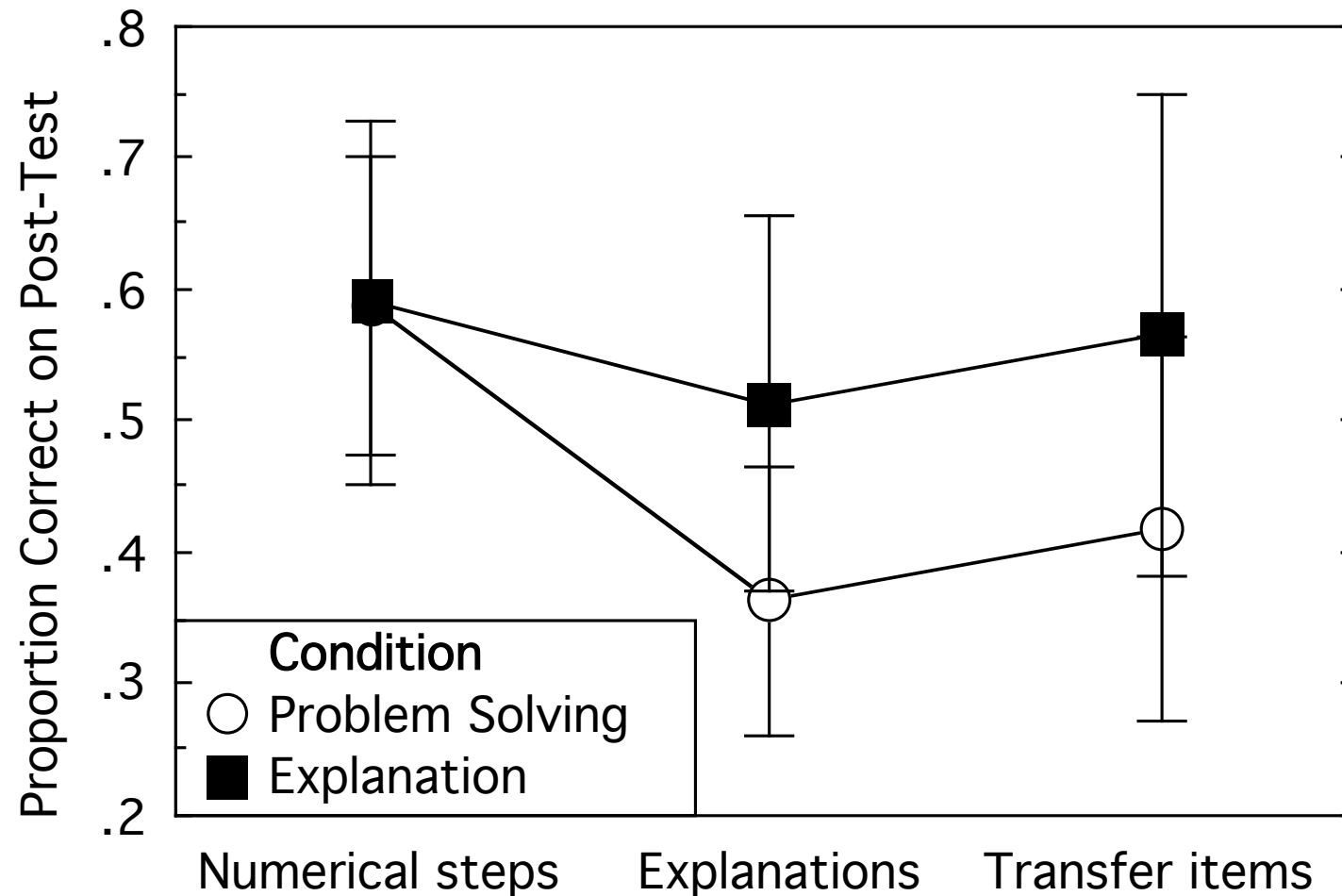


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Problem
solving
answers

Explanation
by reference

Self-explanation yields better transfer



The “*Assistance Dilemma*”

Instructional designer’s dilemma:

When should instruction *provide* students with assistance

vs.

When should it *withhold* assistance & elicit student knowledge construction?

- Fundamental unsolved problem in the learning sciences
 - Defines a design space for instruction
 - Experiments & theory to find areas of space where robust learning is maximized

Assistance Dilemma: Whether to give or to receive?

	Benefits	Costs
Giving information or assistance	Efficiency of communication	Shallow processing Does not engage LTM
Withholding information or assistance	Generation effect Engages & structures LTM	Cost of errors Floundering, confusion, wasted time

The no pain, no gain argument => more assistance

Many researchers & results argue for
lower assistance

place greater demands on students

- “Desirable difficulties” (Bjork)
 - Interleaved practice, delayed feedback, wide spacing
- Abstract examples (Kaminski & Sloutsky)
- Prompting for self-explanation
- Invention as preparation (Schwartz)

Testing effect

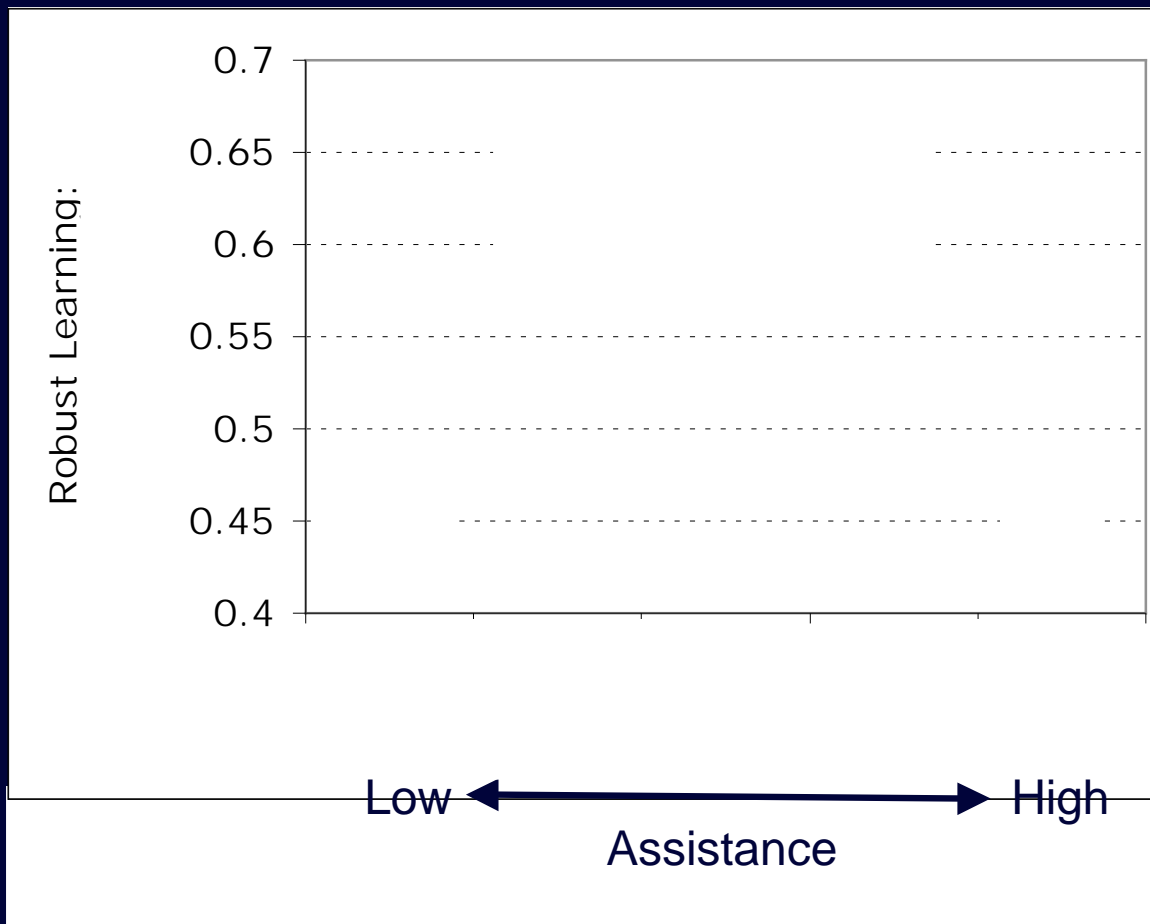
- “Tests enhance later retention more than additional study of the material, even when tests are given without feedback.

This surprising phenomenon is called the *testing effect*”

– Roediger & Karpicke, 2006

Testing Effect Example

Karpicke & Roediger, 06



Memory task:
Study & recall
a list of 40
words

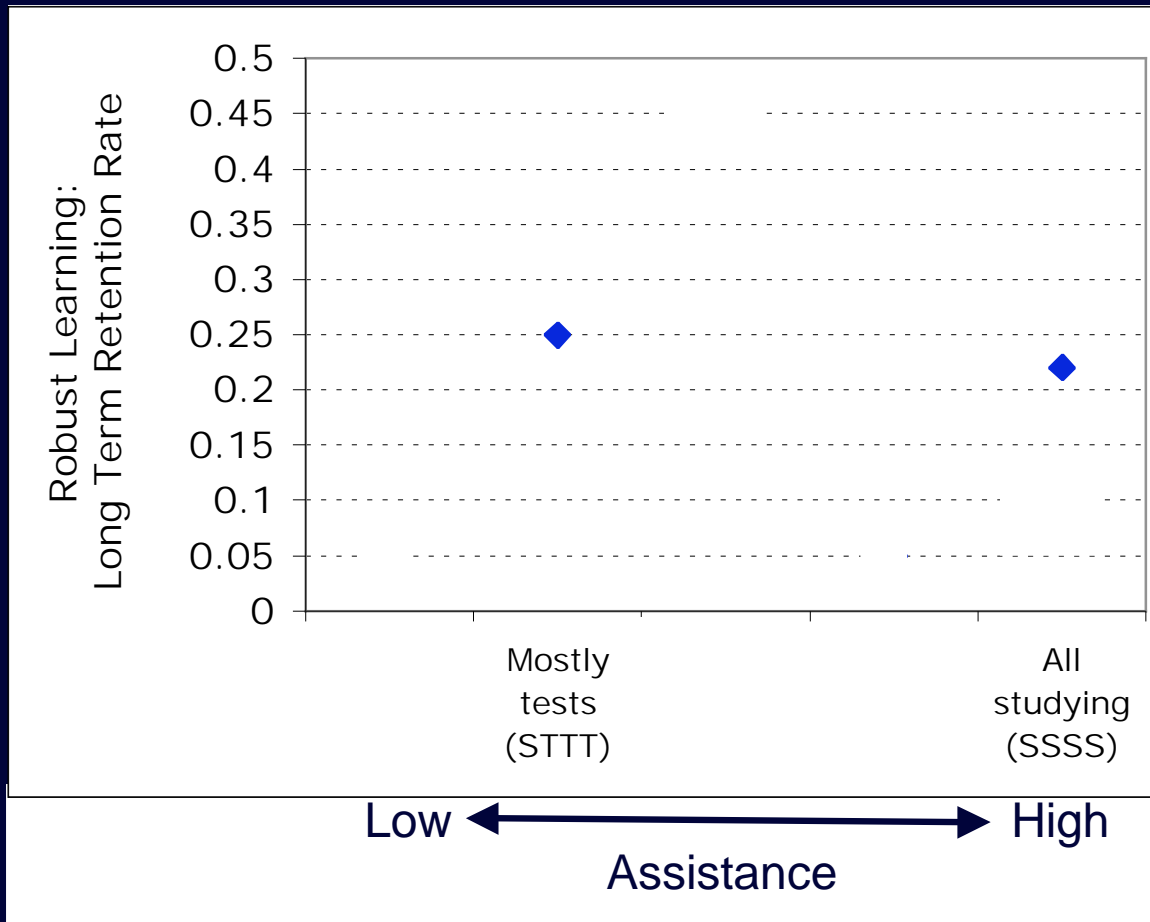
- "Tests enhance later retention more than additional study of the material" (R&K, 06)

Yes

and no

Testing Effect Example 2

Thompson, Wenger, & Bartling (1978)



Memory task:
Study & recall
a list of 40
words

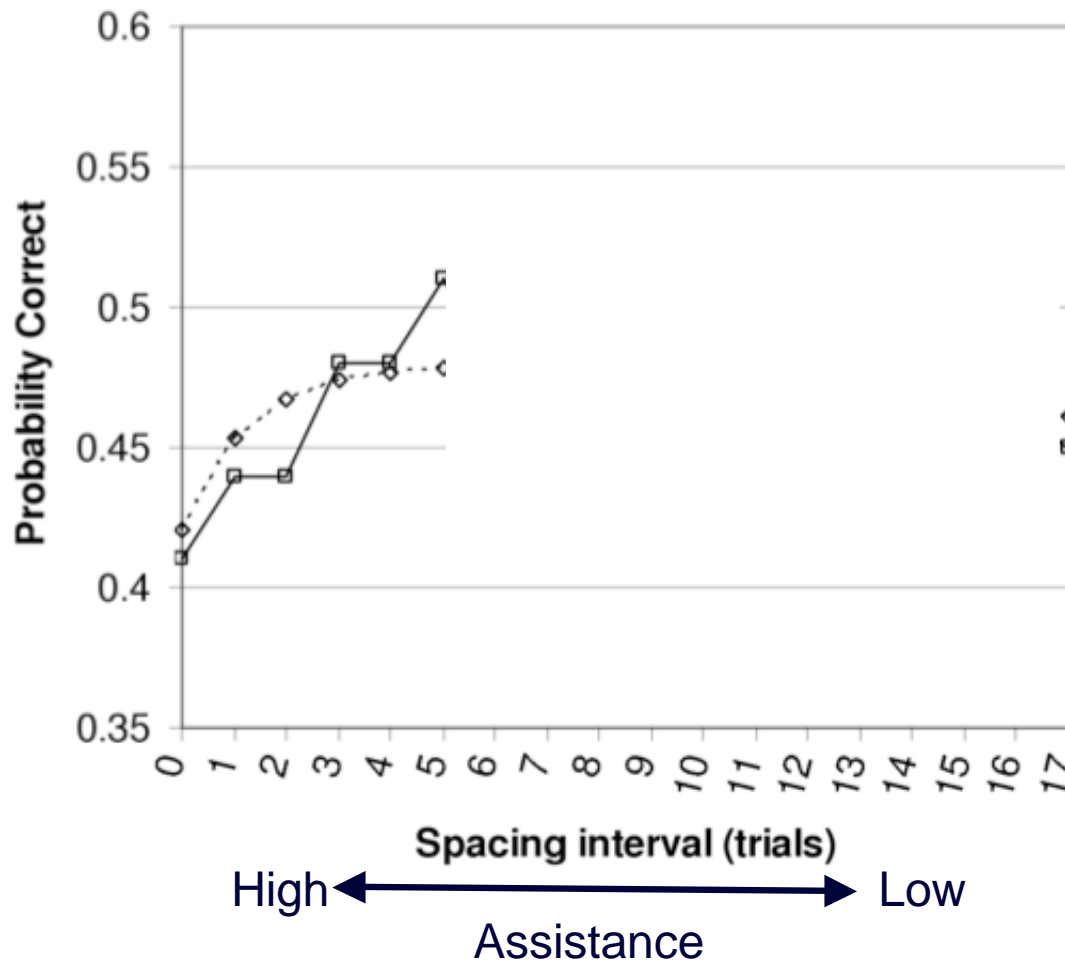
F trials were
tests where
failure yielded
feedback: a
study trial

- "Tests enhance later retention more than additional study of the material" (R&K, 06)

Yes?

and no

Spacing Effect: Wider is better (lower assistance) ...



But too wide yields poorer long-term retention & (not shown) re-study requires more instructional time

Is it "no pain, no gain"?

Or "less pain, more gain"?

- Arguments for higher assistance ...

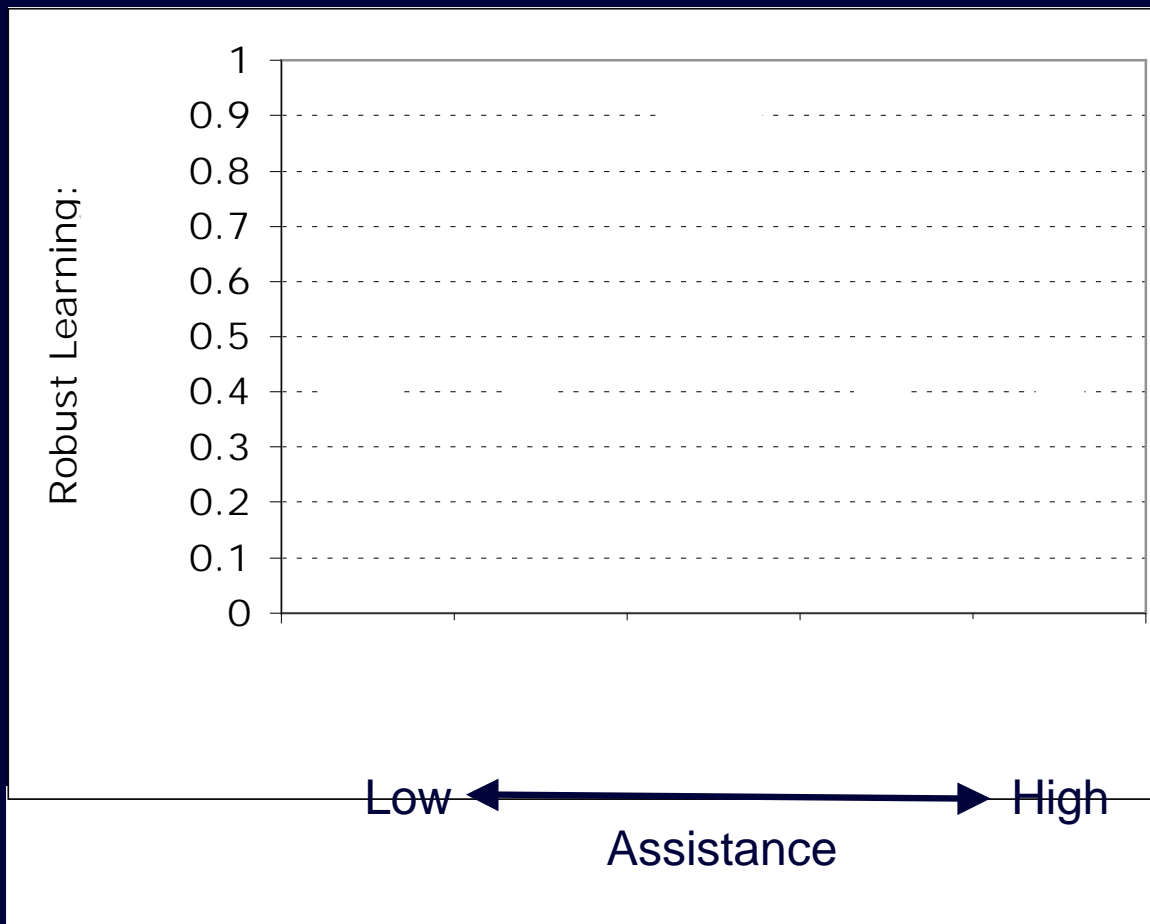
The less pain, more gain argument => more assistance

Other researchers & results argue for *higher assistance*

- Direct instruction (Behaviorist, Klahr, ...)
- Cognitive Load Theory (Sweller, Mayer ...)
 - Reduce “extraneous” cognitive load
 - Mayer’s multimedia principles
 - Modality, contiguity principles
 - Sweller et al.
 - No-goal problems, worked examples ...
- Provocative paper:
 - Why minimal guidance during instruction does not work: An analysis of the failure of constructivist, discovery, problem-based, ... (Kirschner, Sweller, & Clark, 2006).

Worked Examples

Sweller & Cooper



Comparison

- 8 Problems
Solve $(a+b)/c = d$
for a

Vs.

- Example-
problem pairs
Example:

$$\begin{aligned}(a+b)/c &= d \\ a+b &= dc \\ a &= dc - b\end{aligned}$$

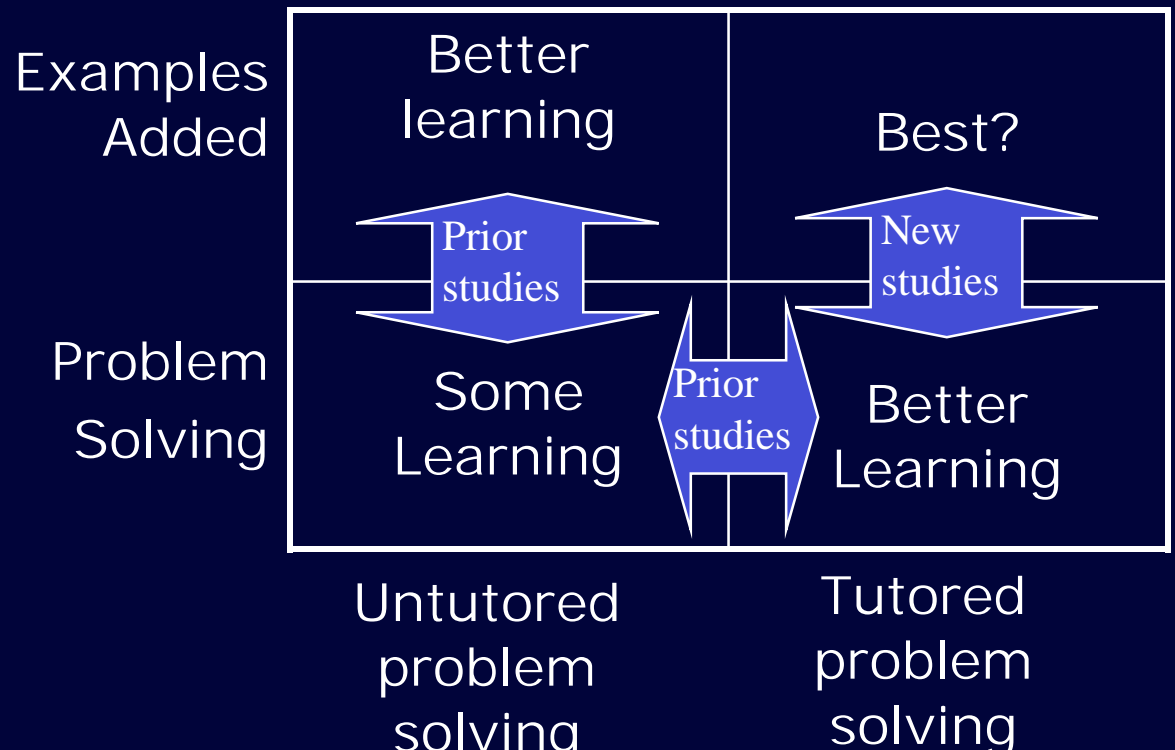
- "a worked example constitutes the epitome of strongly guided instruction" (K, S, C, 06)

Yes

& maybe?

Can worked example principle beat a strong control?

- Prior work: Added worked examples to *untutored* problem solving
- New: Add examples to *tutored* problem solving



PSLC *In Vivo* Studies of Worked Examples in Intelligent Tutors

- Chemistry tutor studies
 - Replacing half problems with worked examples yields more efficient learning -- same outcome in ~20% less time
- Algebra
 - Less time, better long-term retention
- Geometry tutor studies
 - Not only less time, but better conceptual transfer

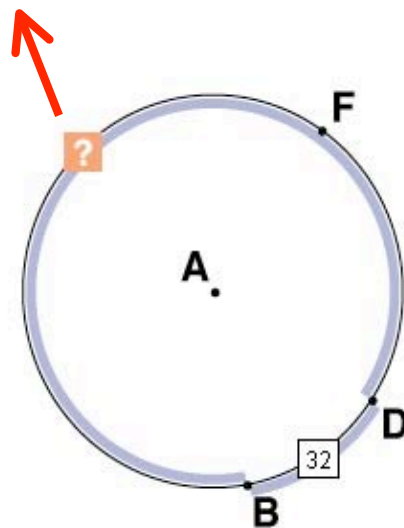
DIAGRAM

Given is circle A with arc BD.

If the measure of arc BD is 32° , what is the measure of arc BFD?

m Arc BFD =

Rule =



Ecological Control =
Standard Cognitive Tutor
Students solve problems step-
by-step & explain

DIAGRAM

Given is circle A with arc BD.

If the measure of arc BD is 34.7° , what is the measure of arc BFD?

$$m \text{ Arc BFD} + m \text{ Arc BD} = 360 \text{ degrees}$$

$$m \text{ Arc BFD} = 360 \text{ degrees} - m \text{ Arc BD}$$

$$m \text{ Arc BFD} = 360 - 34.7$$

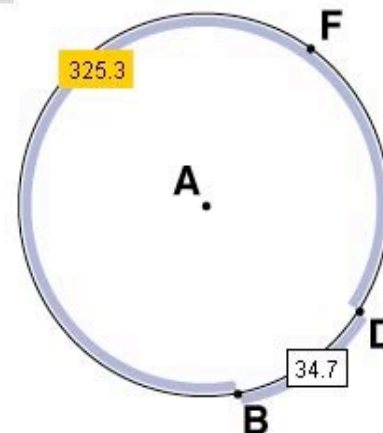
$$m \text{ Arc BFD} = 325.3$$

Rule =

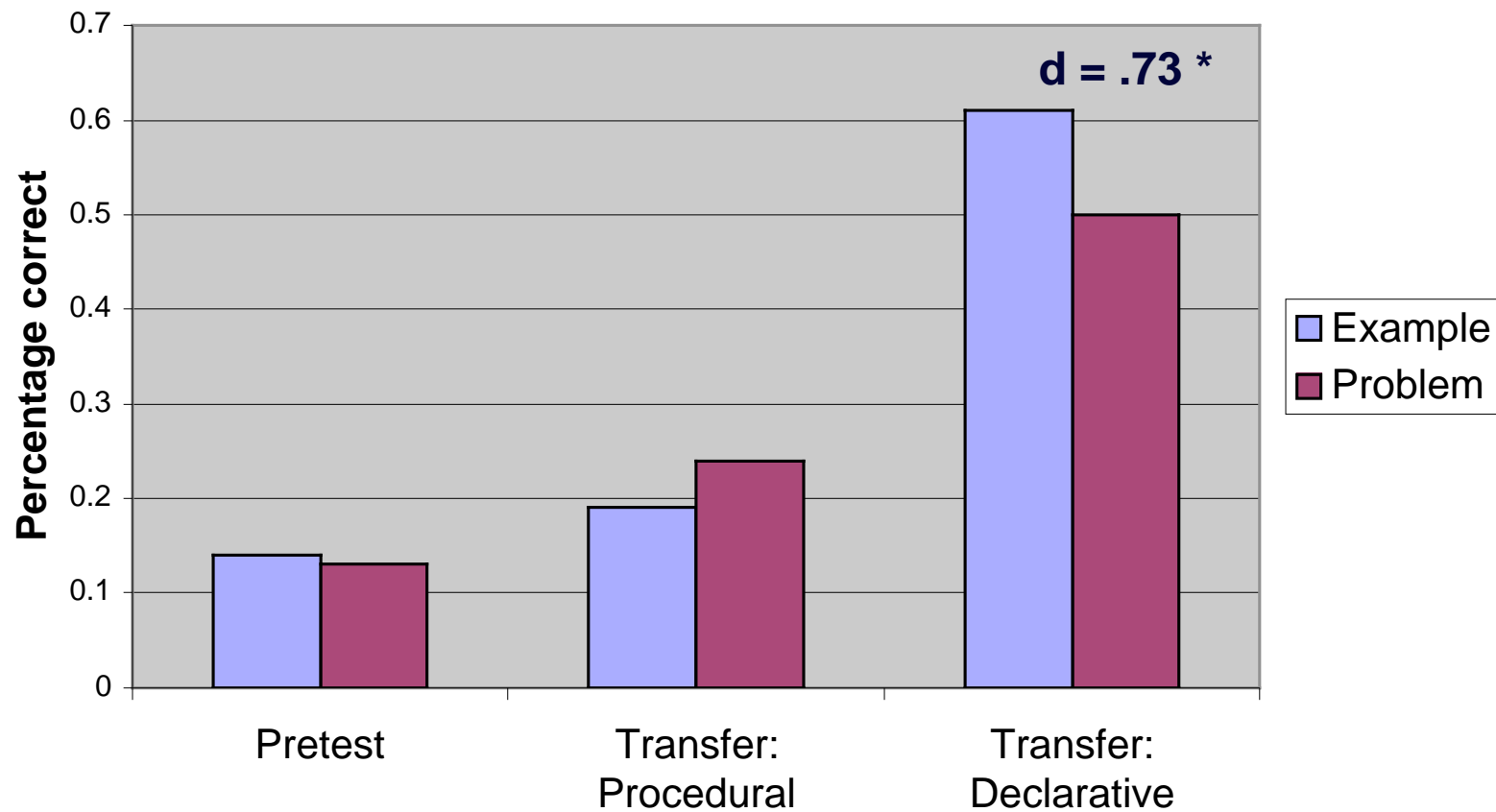
Student still has
to self explain
worked out step

Treatment condition:
Half of steps are given as
examples

Worked out steps with
calculation shown by Tutor



Result: Better conceptual transfer



Think aloud data: Indicate different modes of thinking

- Problem group: Explained how
 - “one can compute the measure of arc EF by subtracting 33.3 from 360”
=> more procedural learning
- Example group: Explained why
 - “This is a major and minor arc, this means the sum of both is 360 degrees”
=> more conceptual principle learning

Theory: Does Cognitive Load Theory Account for Result?

- Tutors are designed to minimize load
 - Tutor provides step-by-step feedback, often gives goal structure to students
 - Mediates goal processing load attributed to problem solving
- Nevertheless students seem to benefit
 - Better to present examples before rather than within problems
- Why?

Why examples before rather than within problems?

- Cognitive Load Theory not adequate
- Alternative: Metacognitive frame of mind evoked by problem solving
 - More engineering than science mode of thinking (Schauble; Miller)
 - More performance oriented than learning oriented (Dweck; Elliot)

Tutor Data Mining Evidence

- What happens when students get a “bottom-out” hint?
- ...

Shih, Koedinger, & Scheines (2008). A response time model for bottom-out hints as worked examples. In *Proceedings of the First International Conference on Educational Data Mining*. [**Conference Best Paper**]

"Bottom-out" hint in Cognitive Tutors => on-demand example

The screenshot displays a software interface for a mathematics tutor. At the top, a green header bar contains the text "8 - Linear Models and Two Quadrant Graphs" and "2 - Graphing with Positive Integer Rates of Change". To the right of the header, the text "_A1 Unit08-2" and "bh1t20" is visible. Below the header, there are navigation buttons: "Look Ahead", "Problems", and "Look Back". To the right of these are buttons for "Solver", "Glossary", "Hint" (with a question mark icon), and "Done" (with a checkmark icon). The main content area is divided into two panes: "Scenario" and "Worksheet". The "Scenario" pane contains a word problem about a rock climber and a list of questions. The "Worksheet" pane shows a graphing area with a grid and a blue line. Overlaid on the "Scenario" pane are four "Hint" windows. The top three hints are partially obscured. The bottom-most hint window is fully visible and contains the text "Enter $2.5t + 67.0$ ". At the bottom of the interface, there are buttons for "Close", "<< Previous Hint", and "Next Hint >>".

8 - Linear Models and Two Quadrant Graphs
2 - Graphing with Positive Integer Rates of Change

_A1 Unit08-2
bh1t20

Look Ahead Problems Look Back

Solver Glossary Hint Done

Scenario Worksheet

A rock climber is currently on the side of a cliff 67 feet off the ground. She can climb on average about two and one feet per

1. When
feet

2.
m
gr

3.
ab
be

4.
fa
wo

To
de
cli
va
her height above the

Hint

Hint

Hint

Enter $2.5t + 67.0$

Close << Previous Hint Next Hint >>

Ground level

Graph

Graph

Graph

Graph

ight above gro

>>

>

Previous Hint Next Hint >>

3 4 5 6 7 8 9

Individual differences in self-explanation

After bottom-out example:

- Some students enter answer & move on
- Others seem to reflect, engage in self-explanation
 - Measure: spend more time than usual
- *Students who reflect more, learn more!*
($R = 0.48$)

A step toward resolving assistance dilemma ... with very broad impact implications!

	<u>Instructor options</u>	
	Give	Elicit
Explicit rules	<i>Lecture</i>	<i>Self-explanation</i>
Implicit: Example solutions	<i>Worked example</i>	<i>Homework</i>

- *Current instruction*: Gives rules & elicits solutions
- *Better instruction*: Gives solutions & elicits rules/concepts

Final Thoughts

- Cognitive Tutors bring learning science to schools, but also ...
- Ed Tech as “Hubble telescope” for learning research!
- Assistance Dilemma is fundamental unsolved problem in learning science
 - Inverted-U function worked out for some dimensions of assistance
 - Using in adaptive systems that optimize for robust learning

END 1

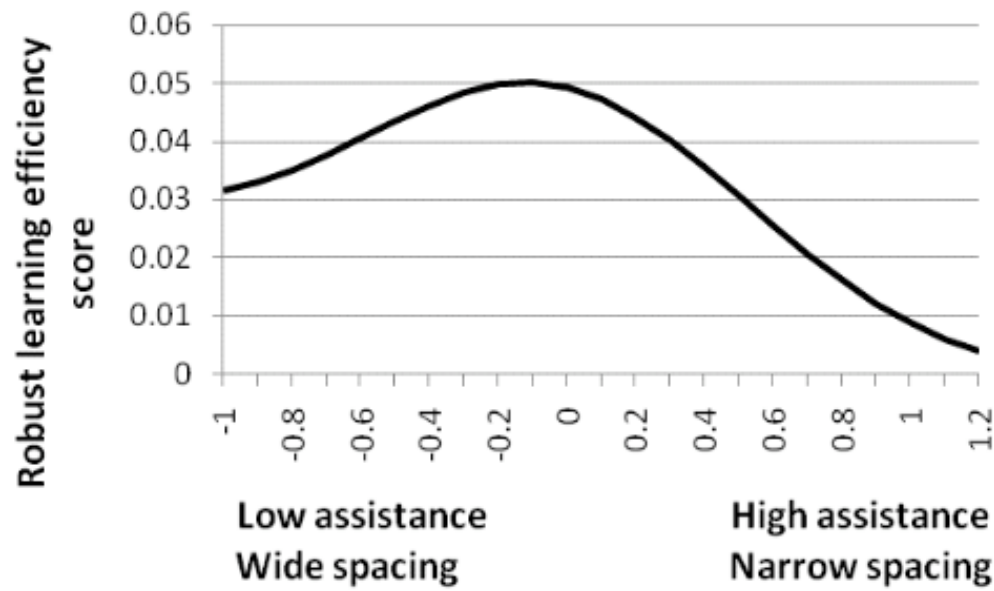
Assistance Dilemma Summary

<u>Instructional support</u>	<i>Poor</i> learning outcome	<i>Good</i> learning outcome
High assistance (less demanding)	crutch	scaffold
Low assistance (more demanding)	undesirable difficulty; extraneous load	desirable difficulty; germane load

Need predictive theory => engineer more effective instruction!

Formulating trade-off => adaptive technology to optimize learning

$$eff_m = \frac{p_m b_{suc} g_m + (1 - p_m) b_{fail} g_m}{p_m (t_m + fixedsucccosts) + (1 - p_m) fixedfailcosts}$$



eff_m = efficiency of robust learning

$p_m * b_{suc} * g_m$ = learning from success

$(1-p_m) * b_{fail} * g_m$ = learning from failure

$p_m (t_m + fsc)$ = success time

$(1-p_m)ffc$ = failure time

m = activation of fact

p_m = probability of recall success

b_{suc} = gain from success

b_{fail} = gain from review after failure

g_m = long-term increase in activation

t_m = time of recall

fsc = time for success

ffc = time for failure

General form of "assistance formula"

For each *learning event*:

Robust learning efficiency gain =

$$\frac{p * \text{benefit-of-success} + (1-p) * \text{benefit-of-failure}}{p * \text{cost-of-success} + (1-p) * \text{cost-of-failure}}$$

p = Probability of success *during* instruction

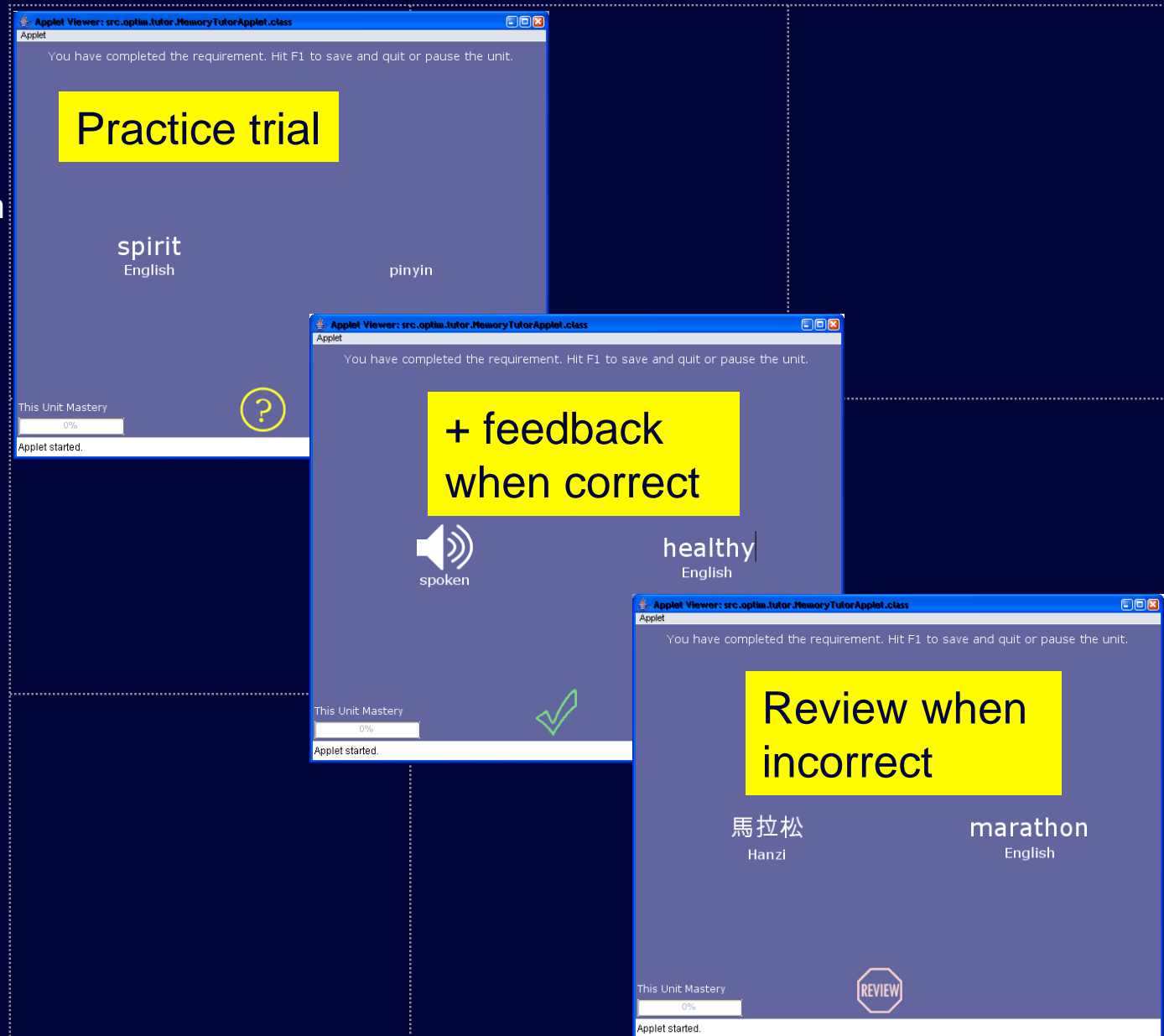
Pavlik's Tutor for Fact Practice

Different
practice
tasks:

English -> Pinyin

Audio -> English

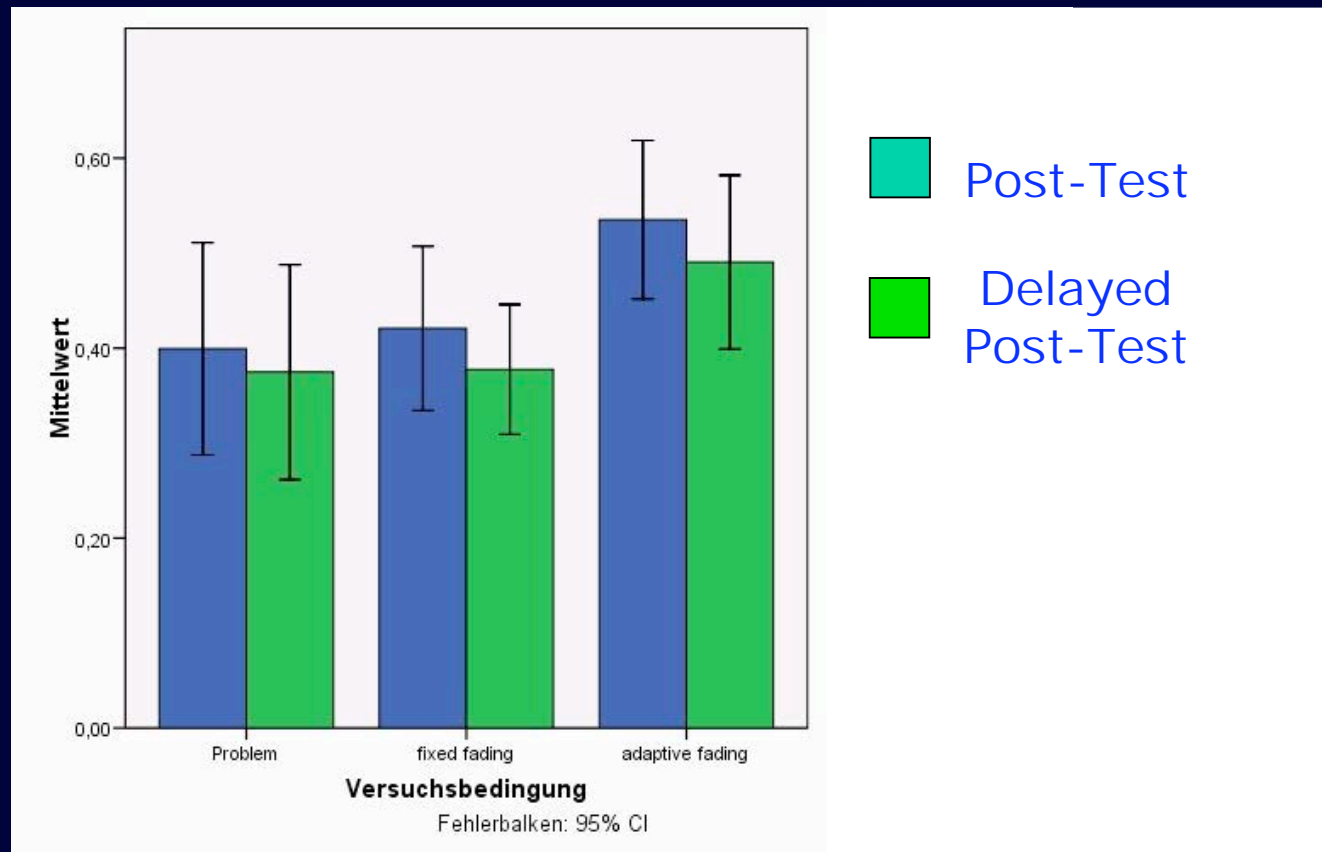
Hanzi -> English



Adaptive Fading of Examples

- Fading based on quality of self explanations of worked out value steps (assessed by Tutor)
- Students who self explain well receive fewer examples than students who self explain poorly

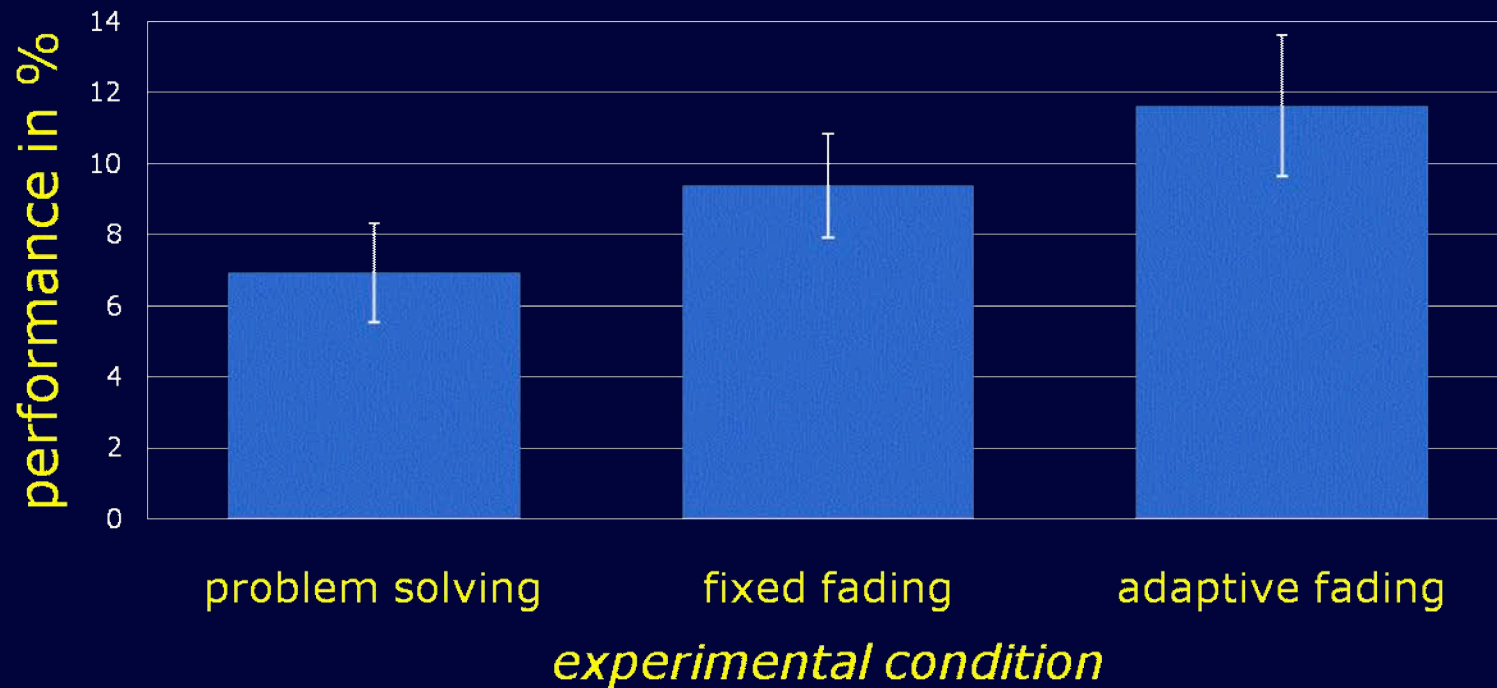
Results



➤ adaptive fading examples > fixed conditions

Results: In Vivo study

Delayed Post-Test



- Result is robust in classroom environment:
adaptive fading examples > problem solving

END 2

Examples < > Study trials

Problems < > Test trials

What's the difference?

- Testing effect studies focus on *facts*
 - Learning process is memory
- Worked example studies are about learning *general rules* & procedures
 - Robust learning requires *category induction* as well as memory

Progress on Principles of Learning to Guide Practice

- PSLC wiki: See learnlab.org
- IES: Cognition Practice Guide
- APS group & web site

Recommendation 7: Help students build explanations by asking and answering deep questions.



When students have acquired a basic set of knowledge about a particular topic of study and are ready to build a more complex understanding of a topic, we recommend that teachers find opportunities to ask questions and model answers to these questions, in order to help students build deep explanations of key concepts. By *deep* explanations we mean explanations that appeal to causal mechanisms, planning, well-reasoned arguments, and logic. Examples of deep explanations include those that inquire about causes and consequences of historical events, motivations of people involved in historical events, scientific evidence for particular theories, and logical justifications for the steps of a mathematical proof.

Examples of the types of questions that prompt deep explanations are *why*, *why-not*, *how*, *what-if*, *how does X compare to Y*, and *what is the evidence for X*? These questions and explanations can occur both during classroom instruction, class discussion, and during independent study.